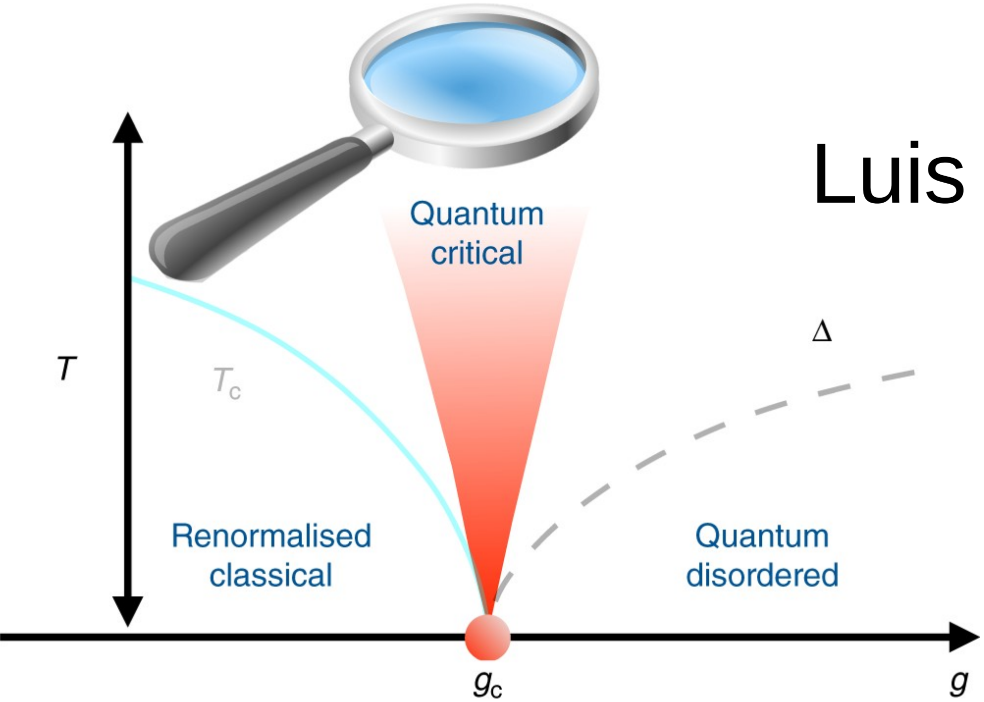
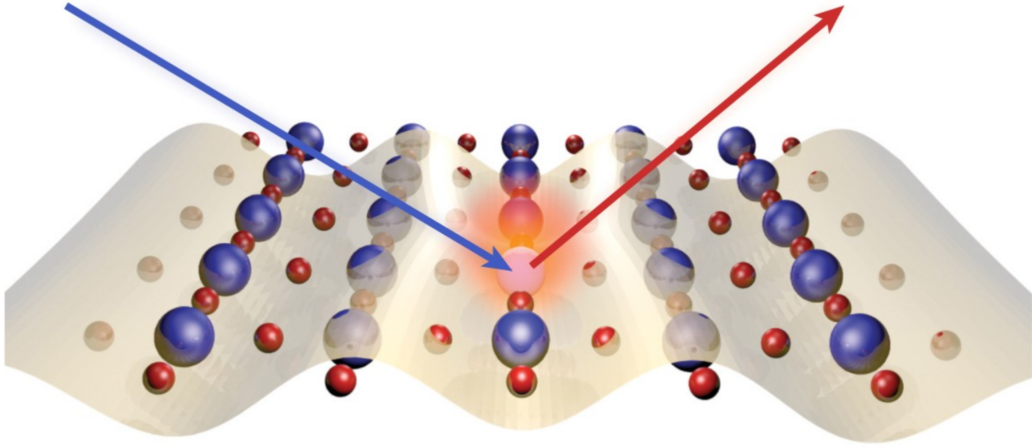
Non-Fermi Liquids: a Brief Introduction to the Confusing World of Strange Metals



Luis Mendoza



Introduction

- Physicists would like to have a theory of metallic behavior
  - Drude Model (1900, classical)
  - Sommerfeld Model (1927, semi-classical)
  - Band Theory (1928, quantum)

Introduction

- Physicists would like to have a theory of metallic behavior
  - Drude Model (1900, classical)
  - Sommerfeld Model (1927, semi-classical)
  - Band Theory (1928, quantum)

All these models treat electrons as free particles, but electrons interact through Coulomb repulsion!

Introduction

- Coulomb potential is not small, $V_C \sim 1/r$
- Why do nearly-free electron theories work at all?

Introduction

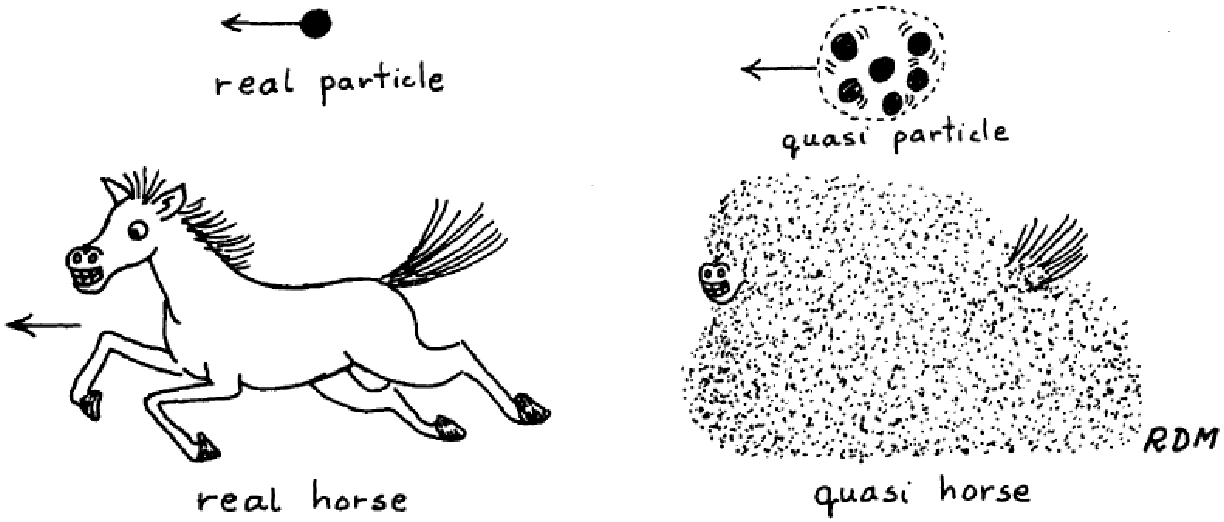
- Coulomb potential is not small, $V_C \sim 1/r$
- Why do nearly-free electron theories work at all?

Fermi Liquid Theory

Due to Landau, describe metal in terms of quasiparticles instead of electrons

Fermi Liquid Theory

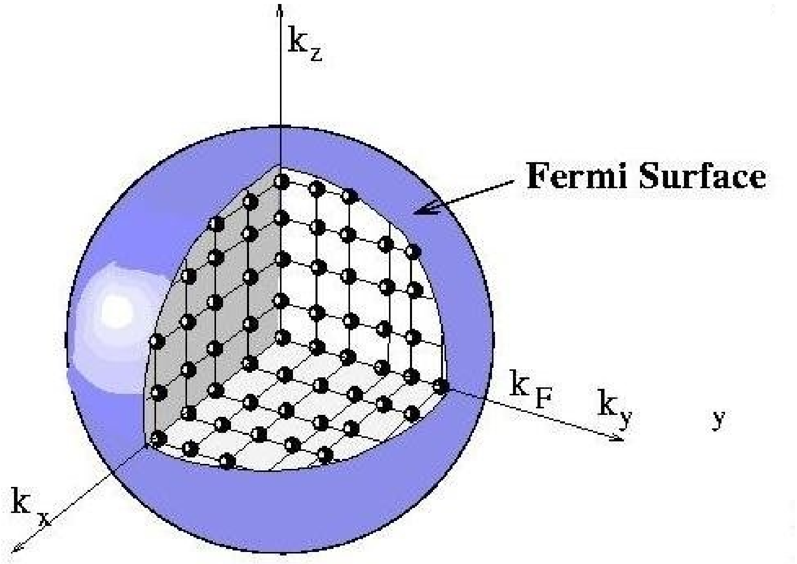
- Quasiparticles have same charge and spin as electrons, but different mass and magnetic moment
- Quasiparticle wavefunctions have a one-to-one correspondence with electron wavefunctions



Fermi Liquid Theory

- We obtain quasiparticles from electrons by turning on interactions very slowly

Quasiparticles are well defined close to the Fermi surface



Fermi Liquid Theory

- Quasiparticle Energy Functional:

$$E = E_0 + \sum_{k,\sigma} \epsilon_\sigma(k)\delta n_\sigma(k) + \frac{1}{2}\sum_{k,k'}\sum_{\sigma,\sigma'} f_{k\sigma,k'\sigma'}\delta n_\sigma(k)\delta n_{\sigma'}(k') + \cdots$$

Spin Susceptibility: $\chi \sim \chi_0$

Resistivity: $\rho \sim T^2$

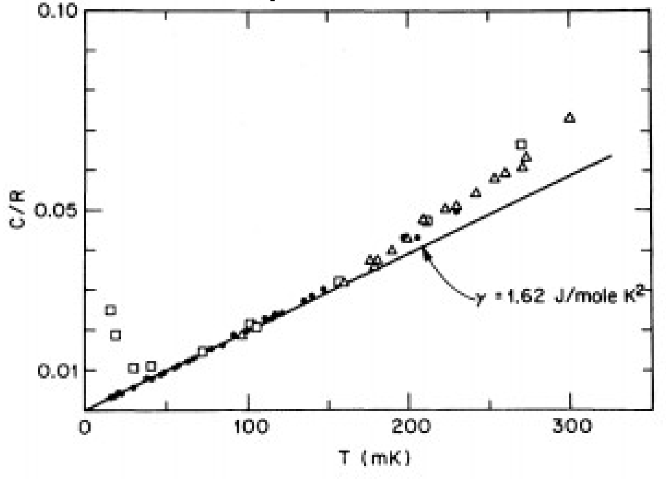
Specific Heat: $C \sim T$

Compressibility: $\kappa \sim \kappa_0$

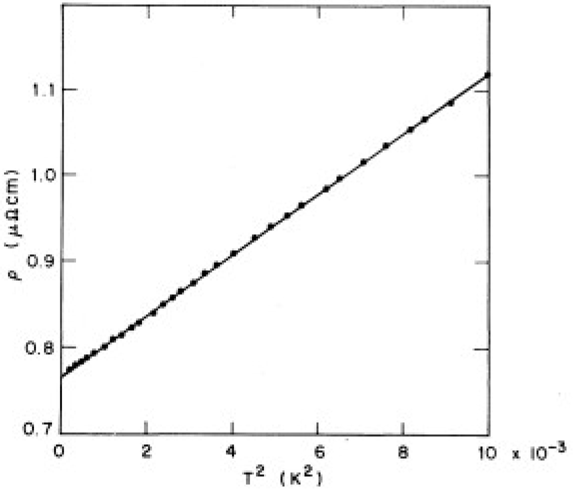
Fermi Liquid Theory

- Great agreement with experiment!

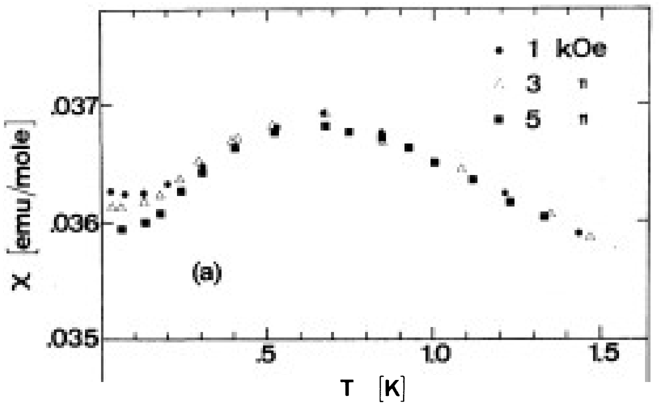
Specific Heat



Resistivity



Spin Susceptibility

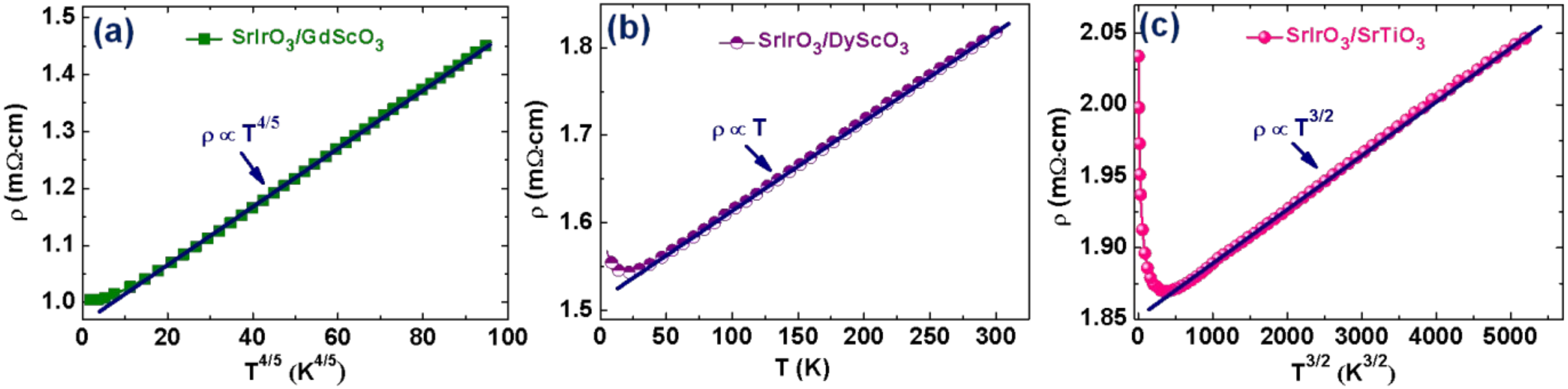


Measurements taken from a $CeAl_3$ sample
(Andres, Graebner, Ott, 1975)

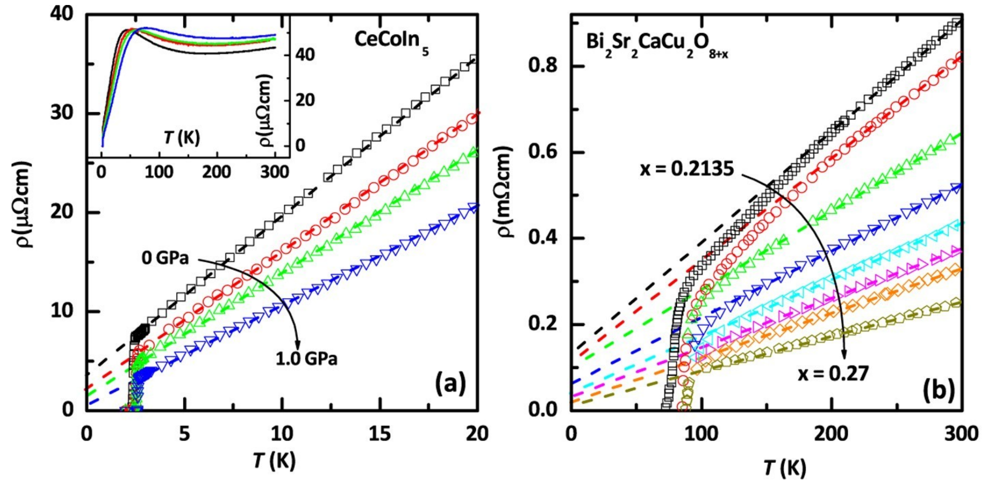
Fermi Liquid Theory Breaks Down

- However, it's not all good news…
- Many systems where Fermi liquid theory breaks down have been discovered since the 1980s
  - Normal state of high temperature cuprate superconductors
  - Rare earth alloys
  - Metals near a quantum critical point
  - One-dimensional Luttinger liquids
  - And many more...

Fermi Liquid Theory Breaks Down



Rare Earth Alloys
(Biswas, Kim, Jeong; 2014)



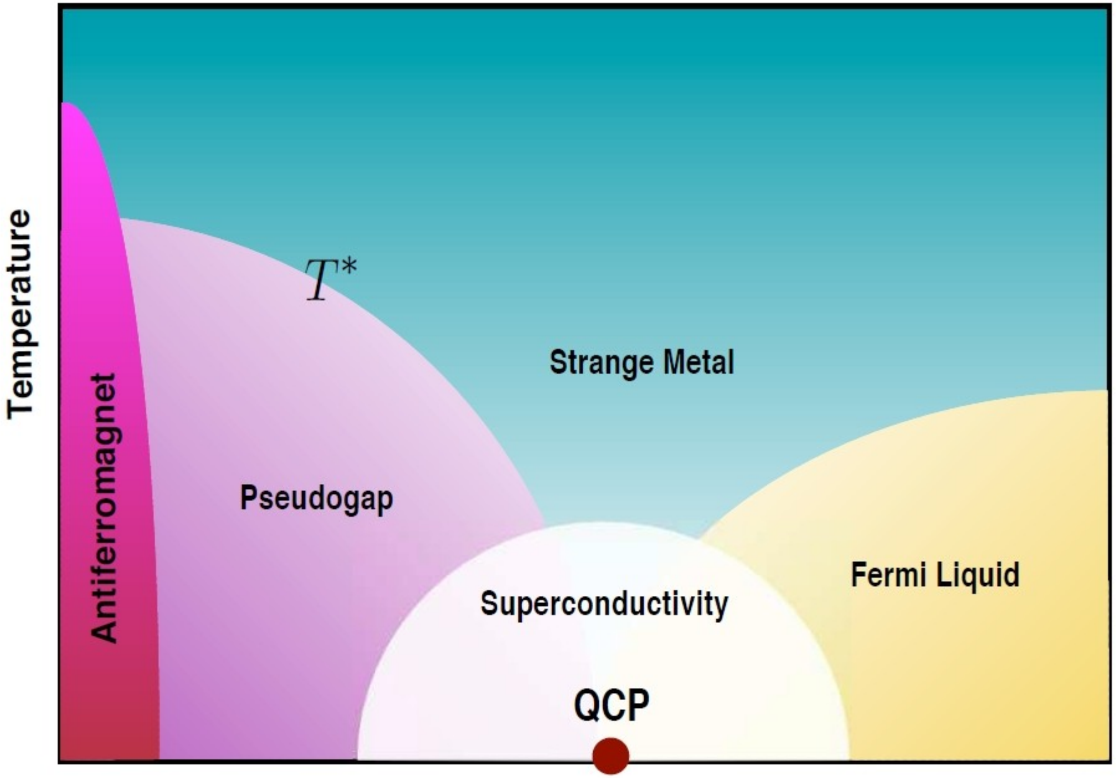
High Temperature Superconductors
(Yu, Liu, Xiao, Yang, 2017)

Fermi Liquid Theory Breaks Down

- Why does Fermi Liquid theory description break down?
- Can we build a model for the behavior of these non-Fermi Liquid systems?

Quantum Critical Metals

Quantum phase transitions occur at very low temperatures and are driven by quantum fluctuations instead of thermal.



Quantum Critical Metals

- If we try to use Fermi liquid framework we find that the effective mass diverges due to fluctuations of quantum order parameter
- Fermi Liquid quasiparticles have zero lifetime and are thus not well-defined
- What can we do?

Quantum Critical Metals

- Quantum Field Theory of course!

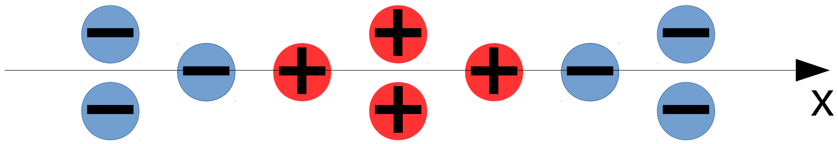
$$L \sim \psi^\dagger(\partial_t - \nabla_i)\psi + g(\psi^\dagger)^2\psi^2 + v_F\psi^\dagger\phi\psi + r\phi^2$$

- Treat order parameter fluctuations as a boson U(1) gauge field ϕ
- Using renormalization group techniques one obtains critical exponents

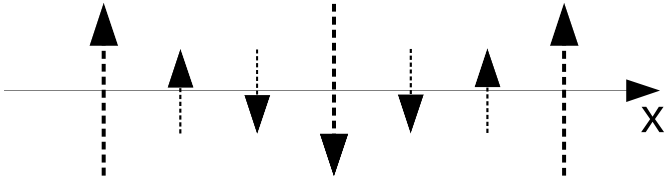
Quantum Critical Metals

- There can be many competing orders when approaching a QCP:

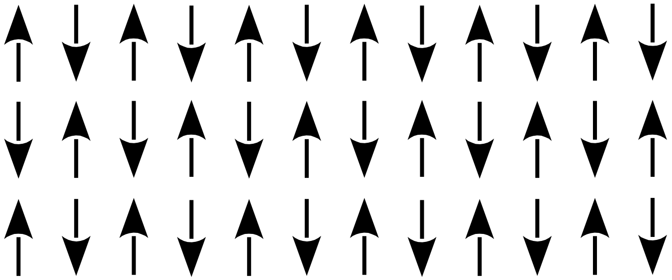
Charge Density Wave



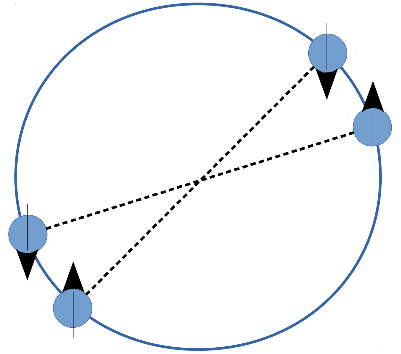
Spin Density Wave



Antiferromagnet

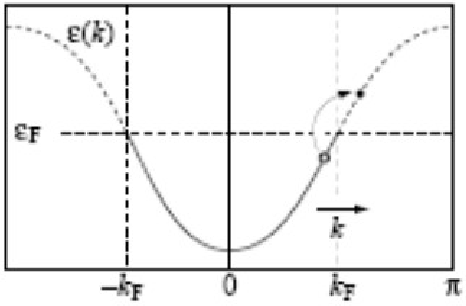


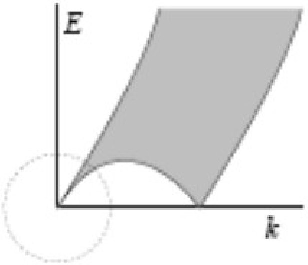
Superconductivity



One Dimensional Metals

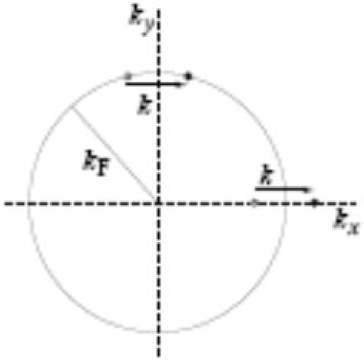
- What's special about one dimension?

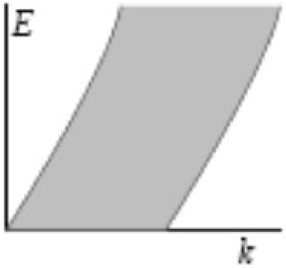




In 1D energy is fully determined by momentum at low energies

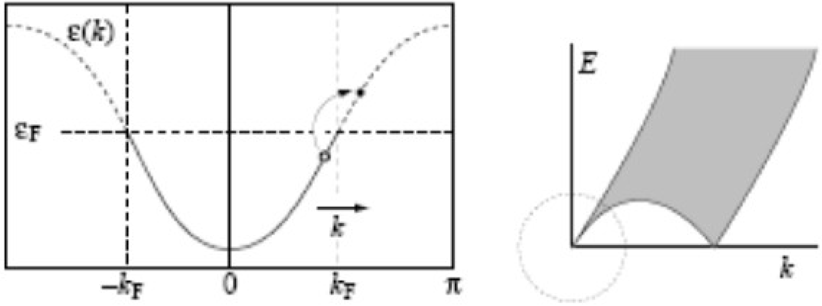
Contrast with 2D case where many different energies are associated with a single momentum value





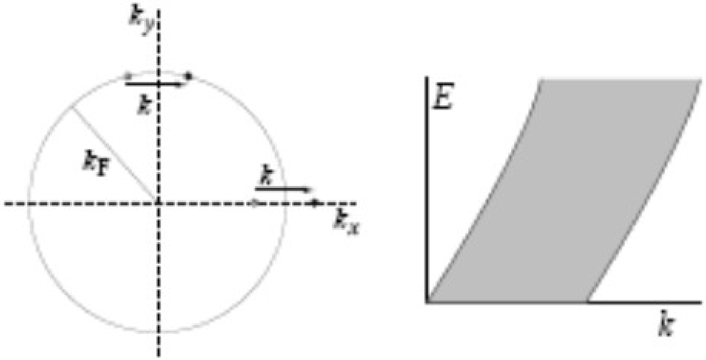
One Dimensional Metals

- What's special about one dimension?



In 1D energy is fully determined by momentum at low energies

Contrast with 2D case where many different energies are associated with a single momentum value



Quasiparticle lifetime in 1D is zero at low energies, Fermi liquid breaks down in 1D

One Dimensional Metals

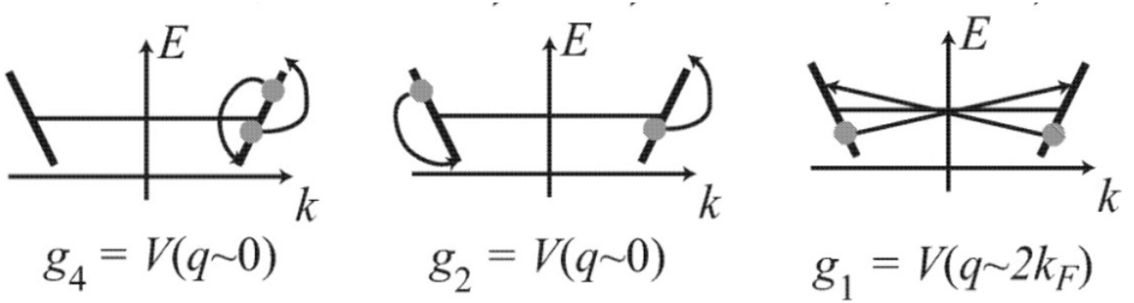
- The free part of the Hamiltonian is

$$H_0 = \sum_k v_F k (c^\dagger_{k,R} c_{k,R} - c^\dagger_{k,L} c_{k,L})$$

- Interaction terms have the general form

$$H_{int} = \sum_{k,k',q} V(q) c^\dagger_{k+q} c^\dagger_{k'-q} c_{k'} c_k$$

- Three dominant interactions at low energies



One Dimensional Metals

- This Hamiltonian can be written in terms of bosonic operators

Total charge density

$$\rho \sim c_{\uparrow}^{\dagger} c_{\uparrow} + c_{\downarrow}^{\dagger} c_{\downarrow}$$

Total spin density

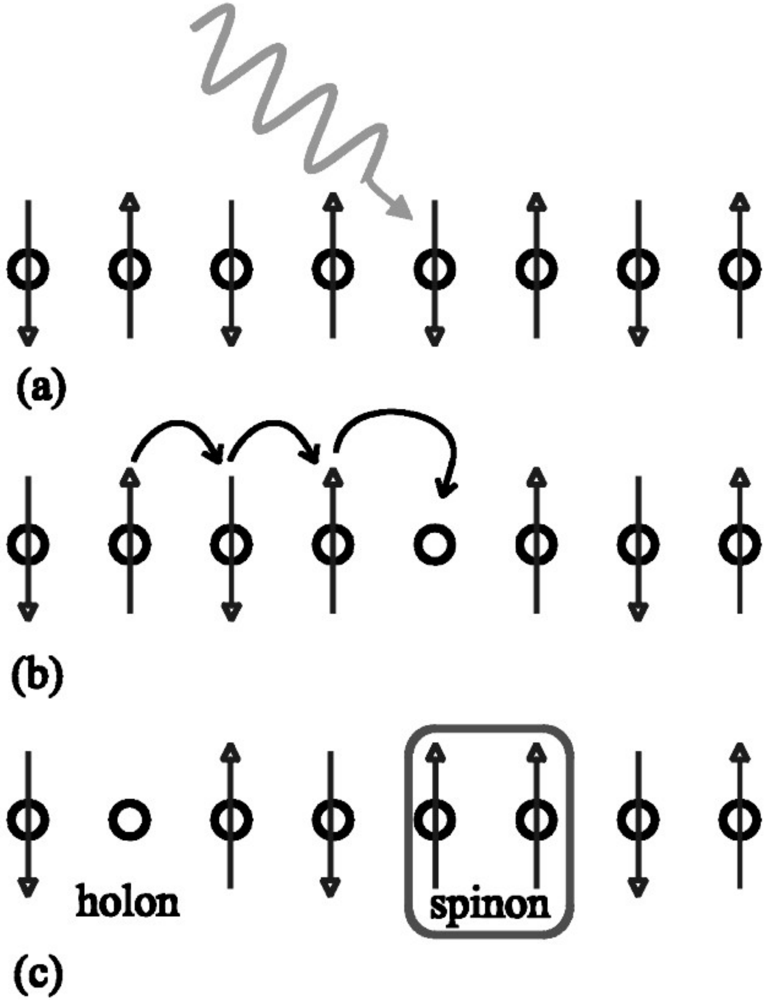
$$\sigma \sim c_{\uparrow}^{\dagger} c_{\uparrow} - c_{\downarrow}^{\dagger} c_{\downarrow}$$

- Using these operators the Hamiltonian factorizes into charge and spin sectors

$$H = H_{charge} + H_{spin}$$

Spin and charge sectors can be diagonalized independently!

One Dimensional Metals



Charge density mode with velocity v_c

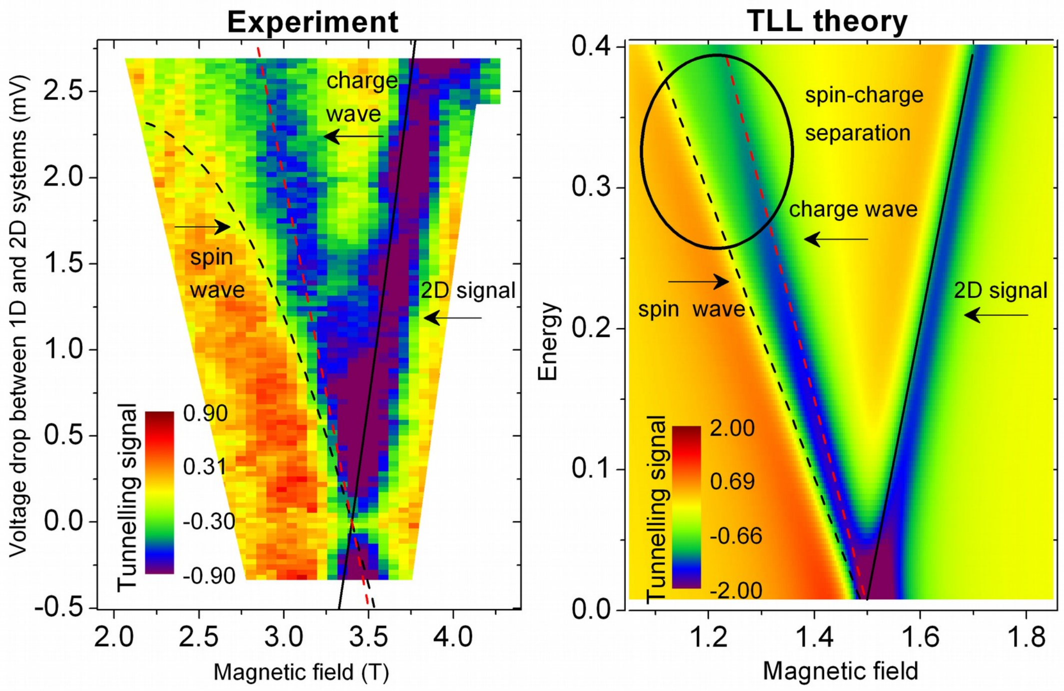
Spin density mode with velocity v_s

In general, $v_c \neq v_s$

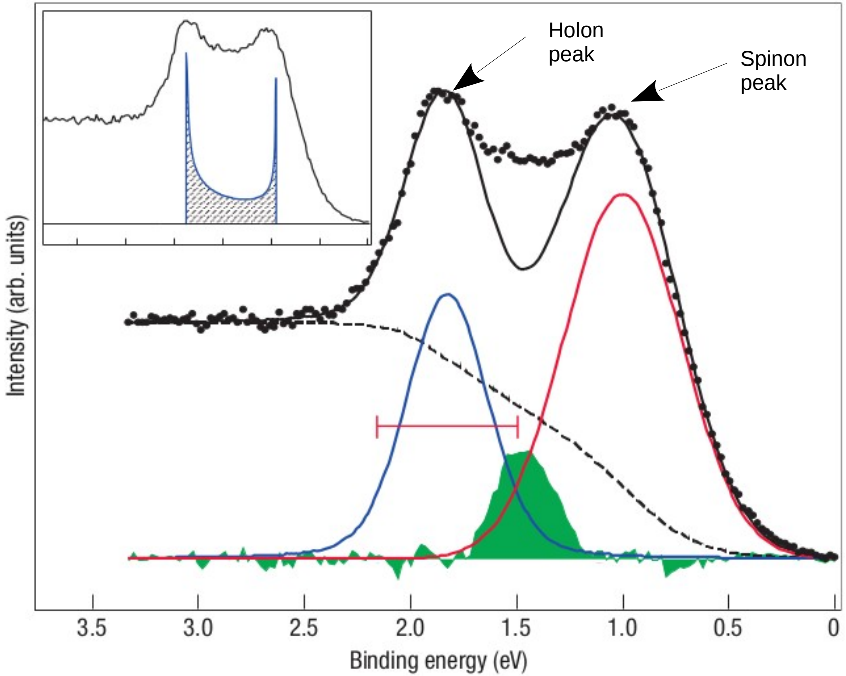
Spin-Charge separation!

One Dimensional Metals

Experimental evidence for spin-charge separation



Direct observation of separate charge and spin waves in the interface of a double quantum well (Jompol, et al; Science 2009)



Spectral function of a $SrCuO_2$ wire (Kim, et al; Nature 2006)

Conclusion

- Strange metallic systems give rise to very interesting collective phenomena
- Very challenging theoretically due to very strong electron-electron interaction, competing orders at low energies, etc
- Studying strange metals could help in fully understanding quantum matter
- There’s a lot of work left to do!