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An introduction to extra dimensions:Kaluza-Klein theories.

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Outline

A brief history of KK theories

Figure: Kaluza and Klein, who in 1921 and 1926 set the foundations of theories with extra dimensions.

The scalar field in 5D

Consider the action of a massless scalar field in 5D:

$$S = \int d^5x \partial_M \Phi(x^\mu, y) \partial^M \Phi(x^\mu, y). \tag{1}$$

Set the extra dimension $x^5 = y$ to be compactified in a circle of radius R, such that $\Phi(x^\mu, y + 2\pi R) = \Phi(x^\mu, y)$. This means that we can perform the decomposition:

$$\Phi(x^\mu, y) = \sum_{n=-\infty}^{\infty} \phi^{(n)}(x^\mu) e^{i\frac{n}{R}y}. \tag{2}$$

Plug this in the action to obtain:

$$S = \int d^4x \left\{ \frac{1}{2} \partial_\mu \phi^{(0)} \partial^\mu \phi^{(0)} + \sum_{n=1}^{\infty} \left[\partial_\mu \phi^{\dagger(n)} \partial^\mu \phi^{(n)} - \frac{n^2}{R^2} \phi^{\dagger(n)} \phi^{(n)} \right] \right\}$$

The vector field in 5D

For a vector field in 5D with a compactified dimension $A_M(x^\mu, y)$ with action

$$S = \int d^4x dy \left[-\frac{1}{4} F_{MN} F^{MN} \right], \tag{3}$$

perform the decomposition:

$$A_M(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_M^{(n)}(x^\mu) e^{i\frac{n}{R}y}. \tag{4}$$

Plug in the action and perform a Gauge transformation to remove the mixed terms:

$$A_\mu^{(n)} \rightarrow A_\mu^{(n)} - i\frac{R}{n}\partial_\mu A_5^{(n)} \qquad A_5^{(n)} \rightarrow 0 \;\; \text{For } n \neq 0 \tag{5}$$

The vector field in 5D

Our action is:

$$S = \int d^4x \left(-\frac{1}{4} F^{(0)}_{\mu\nu} F^{\mu\nu}_{(0)} + \frac{1}{2} \partial_\mu A_5^{(0)} \partial^\mu A_5^{(0)} \right) + \sum_{n \geq 1} \left(-\frac{1}{4} F^{(-n)}_{\mu\nu} F^{\mu\nu}_{(n)} + \frac{1}{2} \frac{n^2}{R^2} A^{(-n)}_\mu A^\mu_{(n)} \right) \tag{6}$$

To match the 5D and the 4D Gauge coupling we can examine the 5D covariant derivative D_μ. Using the expansion of A_M into its KK levels we get:

$$D_\mu = \partial_\mu + i g_5 A_\mu = \partial_\mu + i \frac{g_5}{\sqrt{2\pi R}} A^{(0)}_\mu + \dots \tag{7}$$

The vector field in 5D

The 4D Gauge coupling is then:

$$g_4 = \frac{g_5}{\sqrt{2\pi R}} \tag{8}$$

Note that the coupling has negative mass dimension!

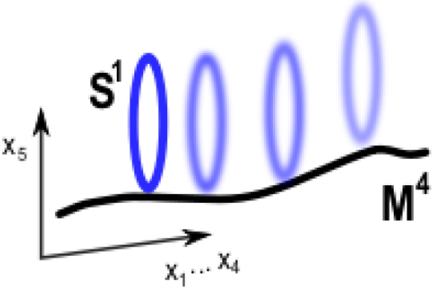


Figure: Example of a 5D spacetime with one dimension compactified in a circle.

Gravitation in KK theories

A 5D graviton G_{MN} decomposes into

$$G_{MN} = \begin{cases} g_{\mu\nu} & \text{A 4D graviton,} \\ g_{\mu 5} & \text{A massless vector,} \\ g_{55} & \text{A real scalar.} \end{cases}$$

The Einstein-Hilbert action in 5D is:

$$S = \int d^5x \sqrt{|G|}^{(5)} R \Longrightarrow^{(5)} R_{MN} = 0. \quad (9)$$

A possible solution is a spacetime of the type $M_4 \times S^1$:

$$ds^2 = W(y)\eta_{\mu\nu}dx^\mu dx^\nu - dy^2. \quad (10)$$

Gravitation in KK theories

Consider excitations in addition to the background metric:

$$G_{MN} = \phi^{-\frac{1}{3}} \begin{pmatrix} (g_{\mu\nu} - \kappa^2 \phi A_\mu A_\nu) & -\kappa\phi A_\mu \\ -\kappa\phi A_\nu & \phi \end{pmatrix}$$

Doing the Fourier expansion and plugging the zero mode part into the Einstein-Hilbert action we get:

$$S = \int d^4x \sqrt{|g|} \left\{ M_{pl}^{2(4)} R - \frac{1}{4}\phi^{(0)} F_{\mu\nu}^{(0)} F_{(0)}^{\mu\nu} + \frac{1}{6} \frac{\partial^\mu \phi^{(0)} \partial_\mu \phi^{(0)}}{(\phi^{(0)})^2} + \ldots \right\}$$

With $M_{pl}^2 = M_*^3 \cdot 2\pi R$. This is the unified theory of gravity, electromagnetism and scalar fields!

- We can generalize to an arbitrary number of extra dimensions: $\bar{M}_4^2 = (2\pi R)^n \bar{M}_{4+n}^{2+n}$. Where $\bar{M}_4 = \frac{M_{pl(4)}}{\sqrt{8\pi}}$.
- This action enjoys the following symmetries:
  1. General 4 dimensional coordinate transformations $x^\mu \longrightarrow x'^\mu(x^\nu)$.
  2. Transformations of the type $y = y' = F(x^\mu, y)$. In this case, in order to leave ds^2 invariant we need:

$$F(x^\mu, y) = y + f(x^\mu) \qquad A_\mu'^{(0)} = A_\mu^{(0)} + \frac{1}{\kappa}\frac{\partial f}{\partial x^\mu}.$$

  3. Overall scaling:

$$y \longrightarrow \lambda y \qquad A_\mu^{(0)} \longrightarrow \lambda A_\mu^{(0)} \qquad \phi^{(0)} \longrightarrow \frac{1}{\lambda^2}\phi^{(0)}.$$

Large extra dimensions

- A broadly use scenario is that our universe is a p brane, which is a surface inside a higher dimensional bulk spacetime.
- We can distinguish different classes of brane world scenarios, but we will mainly talk about one: large extra dimensions.
- In the brane world scenario only gravity feels the extra dimensions.
- Since gravity is so weak, experiments can only test it up to scales larger than 0.1mm.

Large extra dimensions

- This implies that the fundamental scale $\bar{M}_{4+n}^{2+n}$ can be much smaller if we allow the extra dimensions to be large enough.
- If we want the fundamental scale to be of order 1TeV in five dimensions, we would have to require a size of the extra dimension to be of order 10^8km.
- However, starting with six dimensions for a fundamental scale of order 1TeV, we get a size of the extra dimensions of order 0.1mm.
- Notice that this changes the nature of the hierarchy problem!

Some experimetal constraints

How can we test experimentally if extra dimensions actually exists? We have different types of experiments that give us bounds on the size of the extra dimensions in terms of the fundamental scale. We can classify them in:

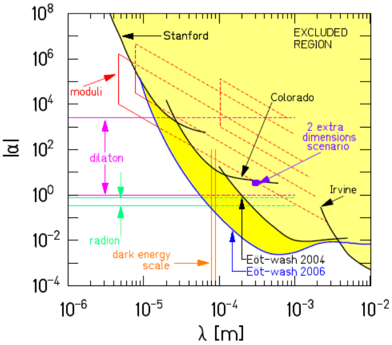
- **Laboratory experiments**: Look for deviations of the classical inverse square law. The bound is $R < 37\mu$m and $\bar{M}_6 > 1.4$TeV.
- **Particle physics experiments**: Experiments in accelerators. Current bounds $\bar{M}_6 > 1$TeV.
- **Astrophysics experiments**: They provide some of the strongest bounds on the large extra dimensions scenario, the strongest being from the low measured luminosities of some pulsars, implying $M_6 > 750$TeV, $M_7 > 35$TeV.

Laboratory bounds

They look for deviations of the usual inverse square law. The experimental test are usually parametrized by the modified potential

$$V(r) = -G_N \frac{m_1 m_2}{r}(1 + \alpha e^{-\frac{r}{\lambda}}) \quad (11)$$

The current bound is $R < 37\mu$m, for a fundamental scale $\bar{M}_6 > 1.4$TeV.



Particle physics constraints

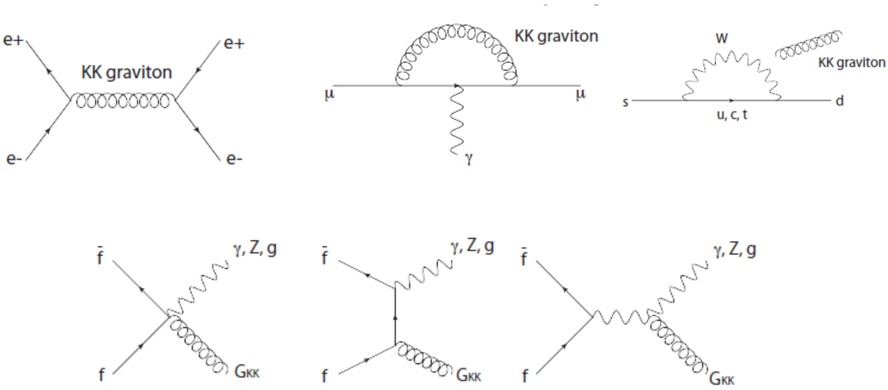


Figure: Feynman diagrams for different interactions including KK gravitons.

Astrophysics Bounds

Astrophysics provides some of the strongest constraints on the large extra dimension scenario:

- **Star cooling**: the rate for KK graviton emission can be estimated as $\Gamma \sim \frac{T^n}{\bar{M}_{4+n}^{n+2}}$. For SN1987A, with $T \sim 30\text{MeV}$ we get $M_6 \geq 14\text{TeV}$ and $M_7 \geq 1.6\text{TeV}$.
- **Diffuse γ ray background from long lived kk graviton decays**: The measurements of EGRET give the following bounds: $M_6 > 38\text{TeV}$ and $M_7 > 4.1\text{TeV}$.
- Limits on gamma rays from neutron stars imply $M_6 > 200\text{TeV}$ and $M_7 > 16\text{TeV}$.
- The low measured luminosities of some pulsars imply $M_6 > 750\text{TeV}$ and $M_7 > 35\text{TeV}$.

References

- F. Quevedo, *Cambridge lectures on Supersymmetry and Extra Dimensions,* ArXiv: hep-th/1011.1491v1.
- Hsin-Chia Cheng, *2009 TASI Lecture – Introduction to Extra Dimensions,* ArXiv: hep-th/1003.1162v1.