Black Holes from the Dawn of Time

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Overview

Friedmann Equations

In General Relativity, the metric is the fundamental dynamical object that describes the gravitational field.

If the spacetime is flat,

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

In a homogeneous, isotropic universe

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]$$

If the universe is filled with a fluid with equation of state $p = w\rho$, then evolution is described by the Friedmann equations ($H = \dot{a}/a$),

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \qquad \dot{\rho} + 3H(\rho + p) = 0$$

History of the Universe

- Planck Era. Quantum Gravity effects are relevant at this stage, so we have no idea what happened.
- Inflation. The universe goes through a phase of accelerated expansion.
- Reheating. The inflaton decays into Standard Model particles and a primordial soup of particles is formed.
- Radiation Domination. Temperature goes down as the Universe expands. Particles become non-relativistic and decouple from the soup.
- Matter Domination. Elements are formed. Photons decouple, so the Universe is not dark anymore. The CMB forms.
- Λ Domination. A second phase of accelerated expansion begins, driven by the Cosmological Constant (Dark Energy?).

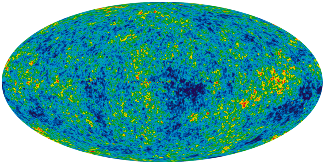
The CMB Puzzles

At early times, the universe was in thermal equilibrium, but expansion caused the temperature to fall, $T \sim a^{-1}$.

Above 1eV, these reactions were taking place,

$$e^{-} + \gamma \leftrightarrow e^{-} + \gamma,$$
$$e^{-} + p^{+} \leftrightarrow H + \gamma.$$

After temperature dropped, electrons recombined and photons decoupled. This is the Cosmic Microwave Background.



There are two puzzling aspects about the CMB,

- The universe is $\sim$ spatially flat, since $k \simeq 0$.
- It's almost completely isotropic.

The metric in conformal time $d\tau = dt/a(t)$ and with $k = 0$ is

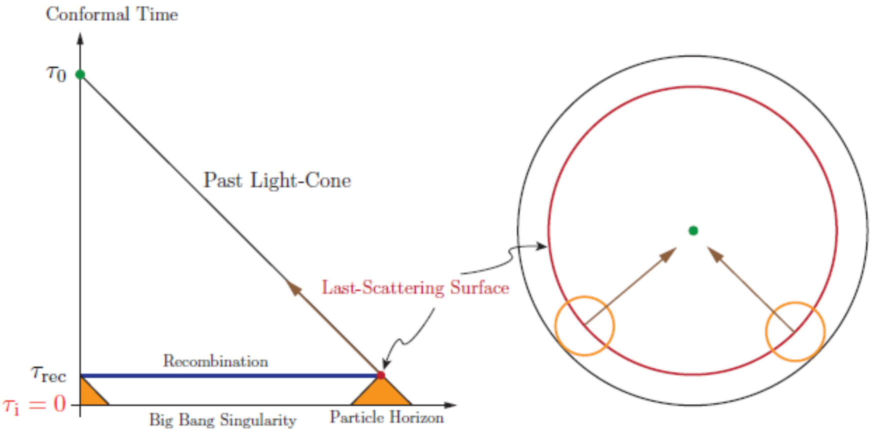
$$ds^2 = a(\tau)^2 \Big[-d\tau^2 + dr^2 + r^2 d\Omega^2 \Big].$$

Photons in distant patches were never in causal contact,

$$\Delta r = \Delta\tau = \int_0^t \frac{dt'}{a(t')}, \qquad \tau \propto a^{\frac{1}{2}(1+3w)}.$$

For $w > -1/3$, the Big Bang is at $\tau = 0 \rightarrow$ No causal contact!

A dynamical explanation is desirable.


Conformal Time
τ_0
Past Light-Cone
Last-Scattering Surface
Recombination
τ_{rec}
$\tau_i = 0$
Big Bang Singularity
Particle Horizon

A Dynamical Solution

Inflation is a phase of **decreasing Hubble horizon**,

$$\frac{d}{dt}(aH)^{-1} < 0 \quad \longrightarrow \quad \epsilon = -\frac{\dot{H}}{H^2} < 1.$$

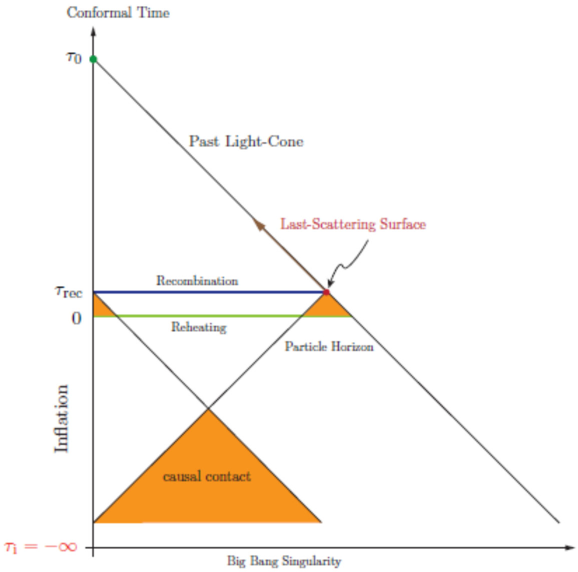
Since $\Omega = \rho\, 8\pi G/(3H^2) \longrightarrow 1$ as $(aH)^{-1}$ decreases

$$\Omega - 1 = k/(aH)^2,$$

the flatness problem is solved. The particle horizon is

$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3w)} \quad \longrightarrow \quad w < -1/3.$$

Thus, $\tau \propto a^{\frac{1}{2}(1+3w)}$, so the Big Bang happens at $\tau = -\infty$ for $w < -1/3$, and the horizon problem is also solved.


Conformal Time
τ_0
Past Light-Cone
Last-Scattering Surface
Recombination
τ_{rec}
0
Reheating
Particle Horizon
Inflation
causal contact
$\tau_i = -\infty$
Big Bang Singularity

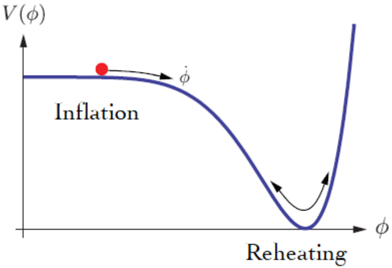
How can we get inflation? The simplest way is through a scalar field[1],

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

If we think of the field as a perfect fluid, it can be shown that

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}.$$

For a very flat potential, $w = -1$ and we get inflation,

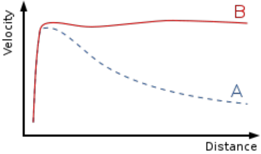


[1] Cosmologists love scalar fields because of homogeneity and isotropy.

The Dark Matter Problem

We have many reasons to believe that there is a type of non-baryonic matter that we cannot see. These are just the tip of the iceberg,

- Galaxy Rotation Curves (A = No DM, B = Observed)



- Gravitational Lensing

- Cosmic Microwave Background

Primordial Black Holes

Any Dark Matter candidate should be...

- Dark (duh),
- Weakly interacting (collisionless),
- Cold (non-relativistic),
- Fluid (non-compact).

There are many ideas: WIMPs, Axions, Sterile ν's, MOND, MACHOs...

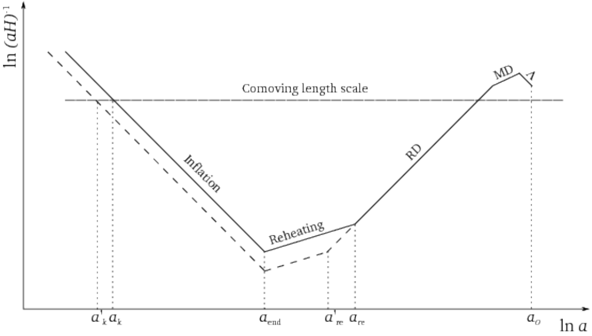
The oldest surviving dark matter candidates are Primordial Black Holes.

Primordial Black Holes are...

Black holes formed in the early universe by mechanisms different to the usual stellar collapse. There are many possibilities, from the collapse of vacuum bubbles and topological defects to inflation.

PBHs from Inflation

For PBHs to form, we need large density fluctuations $\delta = \delta\rho/\rho$. These are produced during inflation.



Density fluctuations occur on a certain scale k (we work with the Fourier transform, δ_k). The scale leaves the horizon, and collapses upon re-entry.

The mass of the PBHs that form is proportional to the total energy in a Hubble patch, and thus depends on the scale of the fluctuation,

$$M_{\rm PBH} = \gamma \frac{4}{3}\pi \rho H^{-3}.$$

with $\gamma \lesssim 1$ because of causality. After a short calculation,

$$M_{\rm PBH} = 10^{18}{\rm g} \left(\frac{\gamma}{0.2}\right) \left(\frac{g(T_f)}{106.75}\right)^{-1/6} \left(\frac{k}{7 \times 10^{13}{\rm Mpc}^{-1}}\right)^{-2}.$$

The fraction of DM in the form of PBHs is ($M_{\rm PBH} = M_{\rm PBH}(k)$)

$$f_{\rm PBH} = \left(\frac{\beta(M_{\rm PBH})}{8 \times 10^{-16}}\right) \left(\frac{\gamma}{0.2}\right)^{3/2} \left(\frac{g(T_f)}{106.75}\right)^{-1/4} \left(\frac{M_{\rm PBH}}{10^{18}{\rm g}}\right)^{-1/2}$$

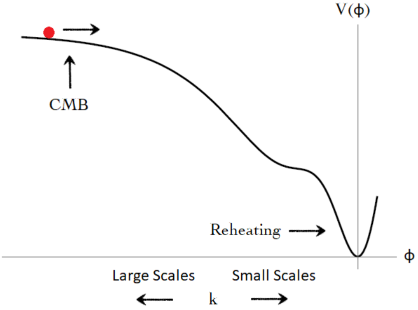
With $\beta(M_{\rm PBH}) \sim \exp[-\delta_c^2/2\mathcal{P}_\delta(k)^2]$ for Gaussian fluctuations. The quantity $\mathcal{P}_\delta(k)$ is known as the power spectrum.

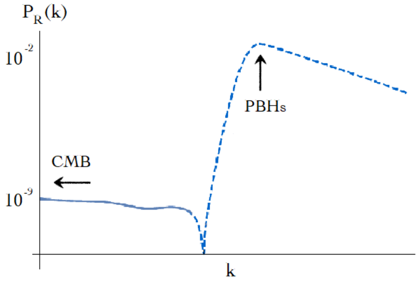
The power spectrum $\mathcal{P}_\delta(\boldsymbol{k})$ tells us how these fluctuations are distributed, can be computed from inflation, and is what CMB experiments measure.

How do we get PBHs from inflation?

Fluctuations produced by inflation at CMB scales are not enough to produce PBHs. We need to enhance the power spectrum at small scales.

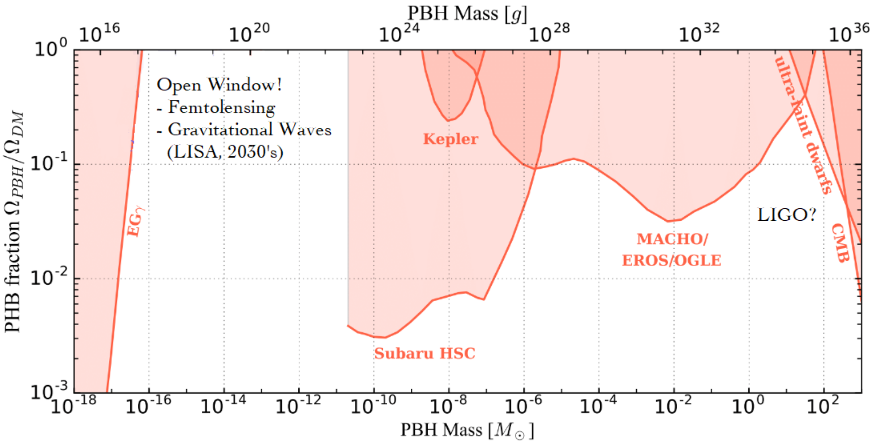
Roughly speaking[2] $\mathcal{P}_\delta(\boldsymbol{k}) \sim H^4/\dot{\phi}^2$,





[2]Subtlety: We don't actually use δ, but $\mathcal{R}$!

Experimental Probes



Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.*[*]
(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equivalent to a significance greater than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$. In the source frame, the initial black hole masses are $36^{+5}_{-4} M_\odot$ and $29^{+4}_{-4} M_\odot$, and the final black hole mass is $62^{+4}_{-4} M_\odot$, with $3.0^{+0.5}_{-0.5} M_\odot c^2$ radiated in gravitational waves. All uncertainties define 90% credible intervals. These observations demonstrate the existence of binary stellar-mass black hole systems. This is the first direct detection of gravitational waves and the first observation of a binary black hole merger.

Some believe that the LIGO detections could be due to PBH mergers, but this zone is ruled out. Perhaps by relaxing some assumptions...

Conclusions

- Inflation provides a simple dynamical solution to the horizon and flatness puzzles of the CMB.
- Primordial Black Holes can account for all the Dark Matter in the Universe.
- PBHs are the only DM candidate that does not require new physics beyond inflation.
- Best of all, it seems that the hypothesis can be decisively tested within the next couple decades through microlensing and gravitational wave experiments.

Thanks for listening!

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  Cosmology and The Physics of Inflation.
  [Lecture Notes]

- A. Guth
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